THE ROYAL AERONAUTICAL SOCIETY

MONTHLY NOTICES,

SEPTEMBER, 1942.

Associate Fellowship Examinations.

Candidates for the December, 1942, examinations are reminded that entries must be received by the Secretary not later than September 30th, 1942.

Election of Members.

The following members have recently been elected:—

Associates.—Charles Ernest Barkworth, Ronald Leonard Biggs, George Simms Bosworth, Sidney Arthur Clark, Harold Clayton, Teodor Dachowski, John Alexander Graham, Francis Henry Henstock, Ernest Frederick Hulse, Duncan Reid Mackay.

Graduates.—James Hodge, Ian Campbell Swan, Robert Thomas Wall (from Student), Reginald Thomas Weatherstone, Ronald White Webster, John William Woodcock.

Companions.—Arthur James Horace Hart, John Jacques Kaye (from Student), Ronald Archibald McAusland.

Students.—Roy Gregory Barnden, Philip Roscoe Best, Richard Peter Boswell, John Tresham Checketts, Una Nellie Gasson, Geoffrey Gould, John Richard Hall, Harold William Harris, Peter Ian Warren Harvey, William Edwin Billing Hurst, Richard Henry Last, Christopher Lawrence Paice, Edwin Douglas Poppleton, Arthur Summers, Eric Edward Wardell.

Journal and Paper Shortage.

Many members of the Society working in official establishments or in firms have opportunities of seeing the Journal apart from the copy to which they are entitled. The severe shortage of the supply of paper is making the printing of the Journal more and more difficult and it has been suggested that those members who can otherwise see the Journal may be prepared to give up their own copy for the period of the war. If a sufficient response is received to the suggestion it will enable the Journal to keep closely to the standard it has set. It may have the further effect of enabling some reduction to be made in the subscriptions of those who do not receive the Journal, though this is a matter for final decision of the Council after the position has been reviewed and cannot be looked upon as a foregone conclusion.

Changes of Address.

Members are urged to notify any change of address at once, as in future JOURNALS which are lost in the post cannot be replaced.

Office Hours.

The Offices and Library are open daily from 9.30-5.0 p.m. on Mondays-Fridays. In the winter months the office will shut one hour before black-out time.

Acknowledgments.

The Council acknowledge with grateful thanks the gift of Journals from Captain A. N. Barrett, Associate Fellow, and from Mr. V. G. Jones, Associate Fellow.

Additions to the Library.

Pamphlets in italics, with location reference following in brackets.
Books marked * or ** may not be taken out on loan.

*B.a.267.—*Focke Wulf F.W. 190/A3 (Engine B.M.W. 801D). Ministry of Aircraft Production Official Report.* (*Stencil.*) 1942. (Y.3.iii.7.)

*B.a.270.—*Avro "Lancaster" Heavy Bomber. Ministry of Aircraft Production Official Report.* (*Stencil.*) 1942. (Y.3.iii. 8.)

B.c.61.—*Gliding and Soaring Simply Explained. By Malcolm Logan. Sir Isaac Pitman and Sons.* 1942. 6*d.* (PB.4/9.)

B.d.3.—Ark Royal. Ministry of Information. H.M.S.O. 1942. 9d.

*B.g.116.—Jane's All the World's Aircraft. 1941 Edition. Edited by L. Bridgman. Samson Low and Marston. 1942. £3 3s. od.

E.b.92.—*Die Berechnung der Tragflächenholme. By Dr. J. Ratzendorfer. Reprint from Oesterreichische Flugzeitschrift.* 1919. (Y.8.a.1.)

E.d.13.—Engineering Drawing. By W. Weeks. Percival Marshall, Ltd. Undated. 2/-.

E.d.14.—How to Read Workshop Drawings. By W. Longland. Percival Marshall, Ltd. Undated. 2/-.

EE.c.63.—Supercharging with Conditioned Air in the Laboratory. By S. D. Heron and others. Aeronautical Research Report No. 24 of the Ethyl Gasoline Corporation. 1938.

G.a.57.—The Damping Capacity of Engineering Materials. By W. H. Hatfield, G. Stanfield and L. Rotherham. E. and F. Spon, Ltd. 1942. (Reprint of Paper read before the N.E. Coast Instn. of Engineers and Shipbuilders, May 15th, 1942.)

G.c.15.—High Temperature Scaling of Exhaust Valve and other Engine Materials. By S. D. Heron. Aeronautical Research Report No. 11 of the Ethyl Gasoline Corporation. 1936.

I.a.28.—Elementary Mathematics. By Hyman Levy. Thos. Nelson and Sons. 1942. 5/-.

I.a.29.—Elementary Mathematics for Wireless Operators. (2nd Edition.) By W. E. Crook. Sir Isaac Pitman and Sons. 1942. 3/6.

L.a.44.—A Primer of Flying. By G. W. Williamson. Sir Isaac Pitman and Sons. 1941. 2/6.

*Q.b.67.—*Aslib Wartime Guides to British Sources of Specialised Information. No. 3: Electrical Engineering and Telecommunications.* (File Case.)

RR.115.—Aeroplane Construction and Operation. By John B. Rathbun. Stanton and Van Vliet, Chicago. 1918.

S.e.42.—Wings Over Olympus. By T. H. Wisdom. Allen and Unwin. 1942. 9/-.

S.e.43.—Blockade by Air. By J. M. Spaight. Geoffrey Bles. 1942. 10/6.

S.e.44.—Sky Sweepers. By Sgt. A. J. Wilson. Jarrolds. 1942. 7/6.

*UU.c.—*National Advisory Committee for Aeronautics, Technical Memoranda*:
No. 1016. Propeller Blade Stresses Caused by Periodic Displacement of the Propeller Shaft. By J. Meyer. (From Luftfahrtforschung, Nov., 1941.)

*WB.21.—Journal and Proceedings of the Royal Society of New South Wales. Vol. LXXV, Part 3. 1941.

*X.a.48.—S.A.E. Handbook, 1942. Society of Automotive Engineers. 1942. $5.

*Z.b.39.—*First Annual International Aeronautical Exhibition, New York City, May* 9-18, 1912. (PZ.2/1.)

J. Laurence Pritchard, *Secretary and Editor.*

THE SHEAR STRESSES IN TAPERED BEAMS.*

By A. G. Pugsley, D.Sc., and R. A. Weatherhead, B.Sc.

Summary.

There is considerable lack of information regarding shear stresses in tapered beams. In this brief report these stresses are examined by a simple extension of the conventional engineering method for uniform beams, and where possible the results are compared with those of classical theory.

Complete agreement is obtained with the few existing classical solutions. The use of the engineering method is illustrated by its application to various types of beam, and curves are drawn illustrating the general nature of the influence of taper on shear stresses.

§1. Introduction.

§1.1. The conventional method of calculating the shear stress in any part of a beam makes no allowance for variation of the beam section, particularly its depth, along the span. Therefore, during some investigations into the failure of a tailplane spar in a rather highly tapered region, consideration was given to the errors arising from the application of the conventional method to a tapered beam. The only relevant published information accessible comprised a footnote in Morley's " Strength of Materials " (1) giving an extension of the conventional engineering method to tapered beams, and some classical results for relevant plane stress conditions which are fully described by Coker and Filon (2). Unfortunately, when applied to the only kind of beam for which they were both suited—*i.e.*, a beam of rectangular section and constant width but tapering depth —these two sources led to results which were definitely at variance. In these circumstances some further exploration of the effect of taper on the shear stresses in a beam appeared desirable.

§1.2. The conventional method derives the shear stress at any point in a given section by considering there the longitudinal equilibrium of that part of a transverse " slice " of the beam which is above the point concerned. Thus in the beam of Figs. 1 and 2, for example, the longitudinal force due to shear stress along the face B_1B_2 is taken as that required to balance the difference of the direct forces on the faces A_1B_1 and A_2B_2. In this way, the engineering method ensures equilibrium but not necessarily compatibility of displacements, whereas the classical theory ensures both; and this distinction might be supposed to account for the discrepancy noted above. On this point it was realised, however, that in a rectangular section beam the direct stresses employed in the engineering method happen to be identical with those of the classical theory, and that for such a beam the two methods should give the same result. A repetition in full of the algebra used in Morley's footnote soon indicated a slip in the final formula given there. It was thus found that the classical and engineering methods give identical results for a rectangular section beam (§3.1).

At this stage Mr. E. H. Atkin kindly provided a classical solution† which he has produced for an I-beam tapering in a particular way, and a further check on the accuracy of the corrected engineering method became possible. For small angles of taper the classical and engineering results again agreed precisely (§4.1).

* Written August 1938.

† Since published in Aircraft Engineering, November, 1938.

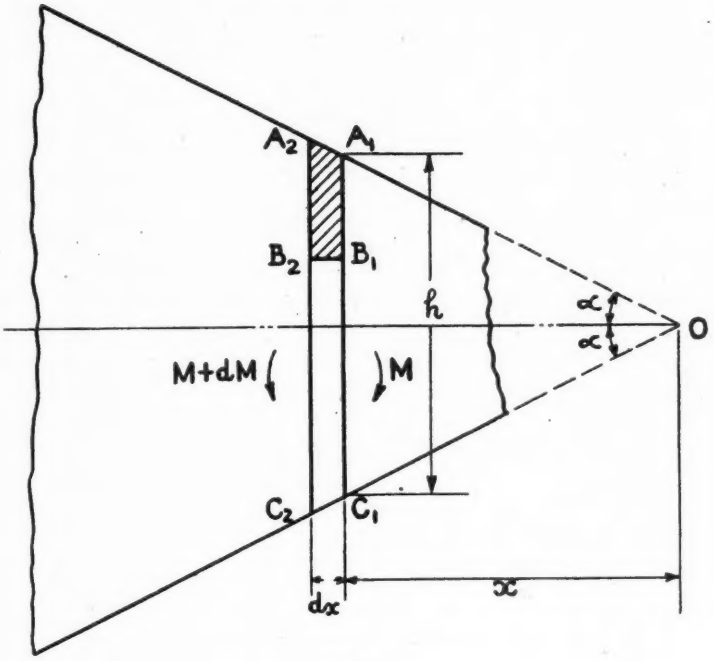


FIG. 1.

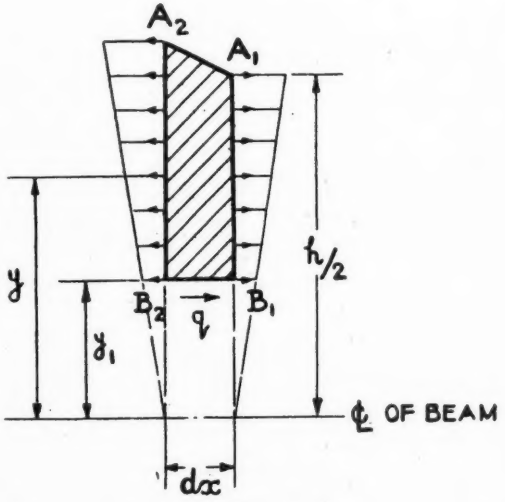


FIG. 2.

Diagrams illustrating notation, etc., for tapered beam of rectangular section.

§1.3. In these circumstances consideration has been given to the more general application of the engineering procedure. It is found that a formula, like that mentioned by Morley, for the shear stress in terms of shear force, bending moment, taper angle, and section inertia, cannot be stated in a convenient general form; and, as the work involved in any particular case is not very heavy, it seems that it will commonly be best to calculate the shear stresses in a tapered beam direct from first principles—in this case the equilibrium equation (1) below.

§1.4. To illustrate the foregoing developments, this short report discusses the shear stresses in three different types of tapered beam—the rectangular section beam, the box or I section beam, and, to indicate the treatment of a beam tapering both in plan and elevation, a beam in the form of a frustrum of a cone. In each case, as approximating to ordinary aeroplane spar conditions, the examples worked relate to a cantilever under uniform spanwise loading.

§2. The Fundamental Equilibrium Condition.

Consider the beam of Fig. 1 in the region of the " slice " $A_1B_1C_1C_2B_2A_2$, a part of which is shown enlarged in Fig. 2. Let the bending moment at the section $A_1B_1C_1$ be M and that at $A_2B_2C_2$ be $(M+dM)$. Then in Fig. 2 the shaded portion $A_1B_1B_2A_2$ of the slice will be in equilibrium under the action of the direct stresses due to M on A_1B_1, those due to $(M+dM)$ on A_2B_2, and the shear stress along the face B_1B_2. Taking q as the intensity of this shear stress and b as the width of the beam, the equilibrium of the portion considered is given by

$$qb_1dx=\int_{y_1}^{(h+dh)/2}\frac{M+dM}{I+dI}\,ybdy-\int_{y_1}^{h/2}\frac{M}{I}\,ybdy, \qquad . \quad . \quad . \quad (1)$$

where h is the total depth of the beam at the section $A_1B_1C_1$,

y_1 is the height of the surface B_1B_2 above the neutral axis,

y is the height of any similar surface in the shaded portion $A_1B_1B_2A_2$,

b_1 is the width of the beam at the height y_1,

I is the moment of inertia of the section $A_1B_1C_1$.

The left hand side of this equation represents the horizontal force due to shear stress on the face B_1B_2, which is balanced by the difference in the horizontal forces, due to direct stresses, on the faces A_2B_2 and A_1B_1 as given by the first and second terms respectively on the right hand side.

§3. Rectangular Section Beam.

§3.1. Comparison of Engineering and Classical Formulæ.

Consider a rectangular section beam of depth h and constant width b with a uniform taper angle α as shown in Fig. 1. For this beam

$$I=\left(\frac{1}{12}\right)bh^3,$$

$$\frac{dh}{dx}=2\tan\alpha,$$

and by substituting these values in (1), integrating and rearranging, we obtain

$$q=\left(\frac{dM}{dx}\right)\left(\frac{6}{bh^3}\right)\left(\frac{h^2}{4}-y_1^2\right)-M\tan\alpha\left(\frac{3}{bh^2}\right)\left(1-\frac{12\,y_1^2}{h^2}\right). \qquad . \quad (2)$$

In this formula the first term on the right hand side is that given in text books for the shear stress in a uniform rectangular section beam due to a shear force dM/dx, and expresses the well-known parabolic distribution of shear stress. The second term, which is zero when the taper angle α is zero, is proportional to the bending moment M and varies parabolically in a given section from a maximum

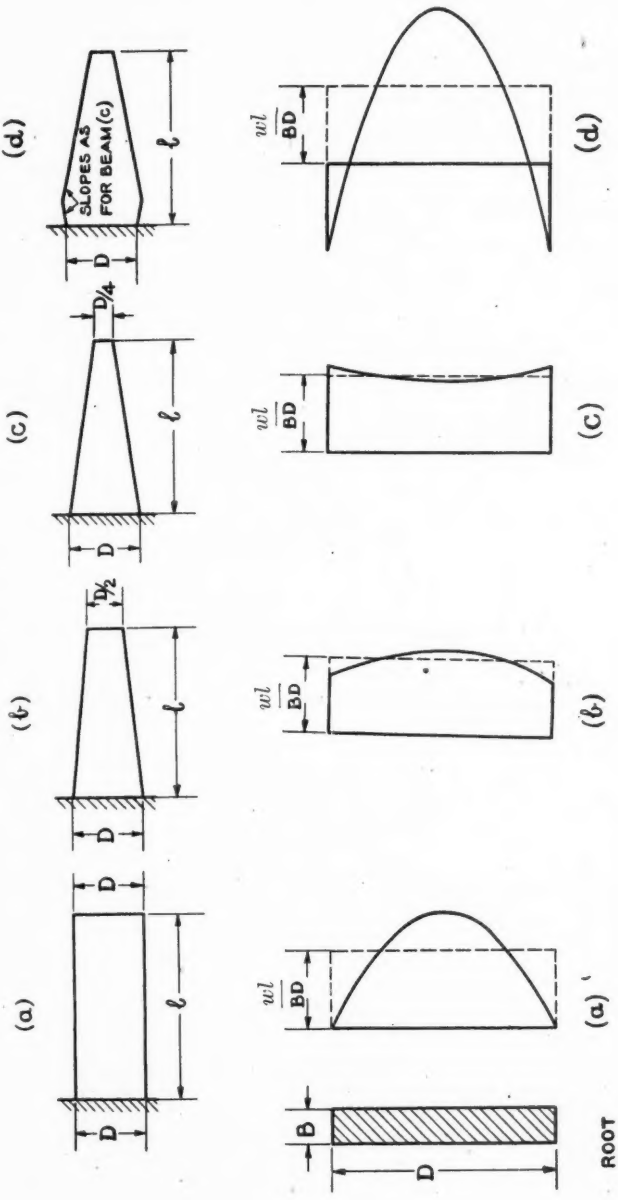


FIG. 3.

Shear stress distributions for rectangular section beams at the root section.

at each surface* through zero at the points $y_1 = \pm h/\sqrt{12}$ to a minimum of opposite sign at the mid depth.

Formula (2) is identical with the result obtained from the classical theory by Coker and Filon (2) for a plane beam with the taper angle α small. A classical solution is also available for larger taper angles, but the formula for q is then a complex trigonometrical one and the corresponding direct stress distribution is no longer strictly linear. In fact, it appears that (2) falls off in accuracy with increase of α at about the same rate as does the ordinary engineering formula for direct stress. Thus when α is 30°, (2) gives a maximum value for q in a practical case about 20 per cent. greater than the true maximum value.

§3.2. Numerical Examples.

Formula (2) has been applied to a uniformly loaded cantilever, of length l and width B, with an overall depth tapering linearly from D at the root to D/n at the tip. The shear stress at the root section of such a cantilever is given by

$$q = \left(\frac{3wl}{nBD^3}\right)\left\{(n+1)\frac{D^2}{4} + (n-3)\,y_1^2\right\} \quad . \quad . \quad . \quad (3)$$

where w is the loading per unit run.

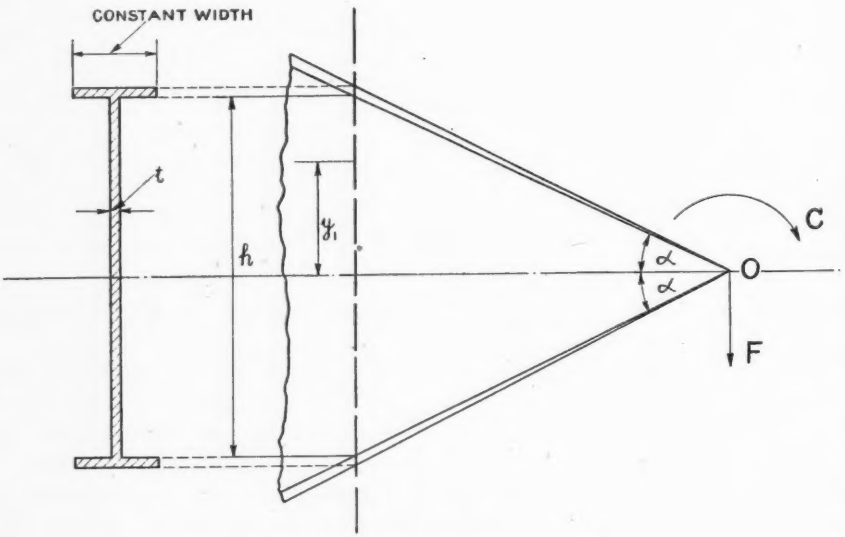


Fig. 4.

Diagram illustrating notation, etc., for tapered beam of I-section.

Fig. 3 shows the distribution of shear stress (3) in terms of the average shear stress wl/BD for the values of $n = 1$, 2 and 4. It will be seen that, although the shear stress distribution changes rapidly with taper, the maximum value of the shear stress for the small practical tapers considered does not exceed that appropriate to zero taper. This result is largely due to the relative signs of the bending moment and shear for such a cantilever; were these changed or the beam taper negative, *i.e.*, taper involving decreasing depth in a region of

*The value of this maximum is necessarily such that the effect of the shear and direct stresses at the surface is a resultant stress acting along the surface.

increasing bending moment, more serious stresses would arise. Thus if for some reason one of the cantilevers were to have a negative taper near its root equal to that of Fig. 3 (*c*), then the distribution of shear stress would change to that of Fig. 3 (*d*).

§4. Box or I-Section Beam.

§4.1. Comparison of Engineering and Classical Formulæ.

The particular form of beam for which a classical solution has been obtained by Mr. Atkin is of the wedge shape shown in Fig. 4. Each flange is of constant width with a depth increasing linearly from zero at the apex o and the web is of uniform thickness t. The only loading on this wedge-shaped cantilever is a shear force F and couple C applied at the apex o.

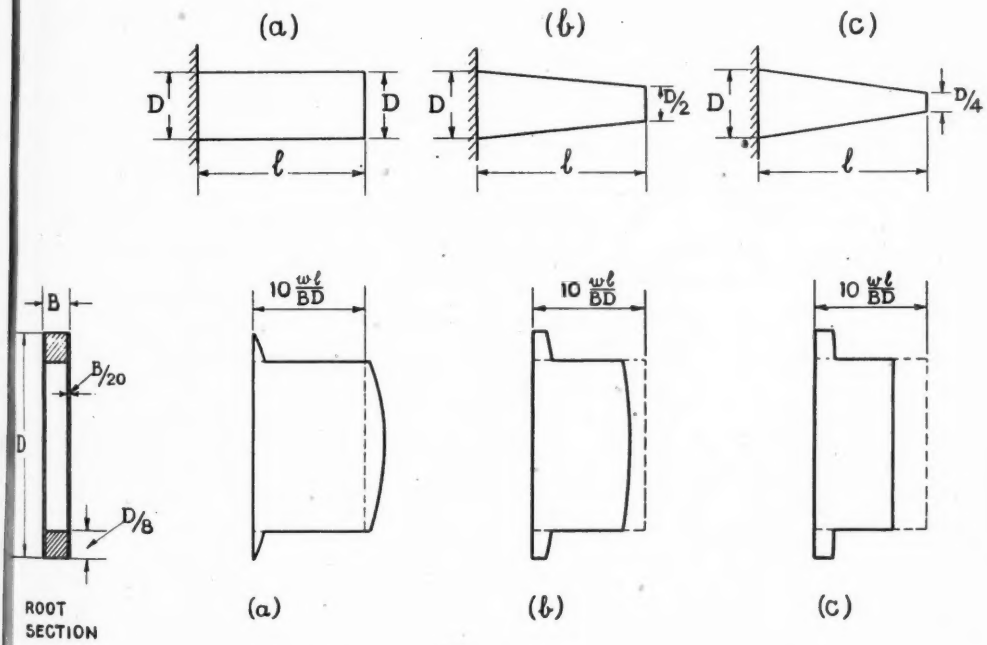


Fig. 5.

Shear stress distributions for box section beams at the root section.

By applying equation (1) to this particular cantilever, taking the flange depth to be negligible compared with the web depth h and the taper angle α to be small, the shear stress in the web at a height y_1 (see Fig. 4) is given by the engineering method as

$$q=\left(\frac{F}{th}\right)\frac{(N+1-y_1^2)}{\{h^2(N+\frac{1}{3})\}}-\left(\frac{C}{th^2}\right)\left\{\frac{(2\tan\alpha)}{(N+\frac{1}{3})}\right\}\left\{(N+\tfrac{1}{2})-6\frac{y_1^2}{h^2}\right\} \quad . \quad (4)$$

where N is the ratio of the area of one flange at a given section to the web area (th) there. This result (4) of course agrees with (2) when the flange area is zero ($N=0$).

Mr. Atkin's analysis by the classical methods for plane stress problems gives a formula for the shear stress q which is identical with (4).

§4.2. Numerical Examples.

A box section beam is chosen as typical of aeroplane spar design in wood. As before, three uniformly loaded cantilevers are considered, each having the same length and the root section shown in Fig. 5, but with the overall depth tapering linearly to tip values of D, $\frac{1}{2}D$ and $\frac{1}{4}D$ respectively, where D is the overall depth at the root section. The spar width and flange depth are kept constant, the flange depth being equal to $\frac{1}{8}D$ so that in the most tapered instance the flanges just occupy the whole of the beam depth at the tip.

By applying formula (1) and noting that the net width of the section changes from B across the flange to $B/10$ across the web (Fig. 5), the shear stress in the flange at the root section is found to be

$$q=\left(\frac{wl}{nBD^3}\right)\{(1.98\,n+0.45)\,D^2+(1.78\,n-11.47)\,y_1^2\} \quad . \quad (5)$$

where as before the tip section depth is $1/n$ times that at the root. The shear stress in the web at the root is

$$q=\left(\frac{wl}{nBD^3}\right)\{(5.58\,n+6.25)\,D^2+(1.78\,n-11.47)\,y_1^2\} \quad . \quad . \quad (6)$$

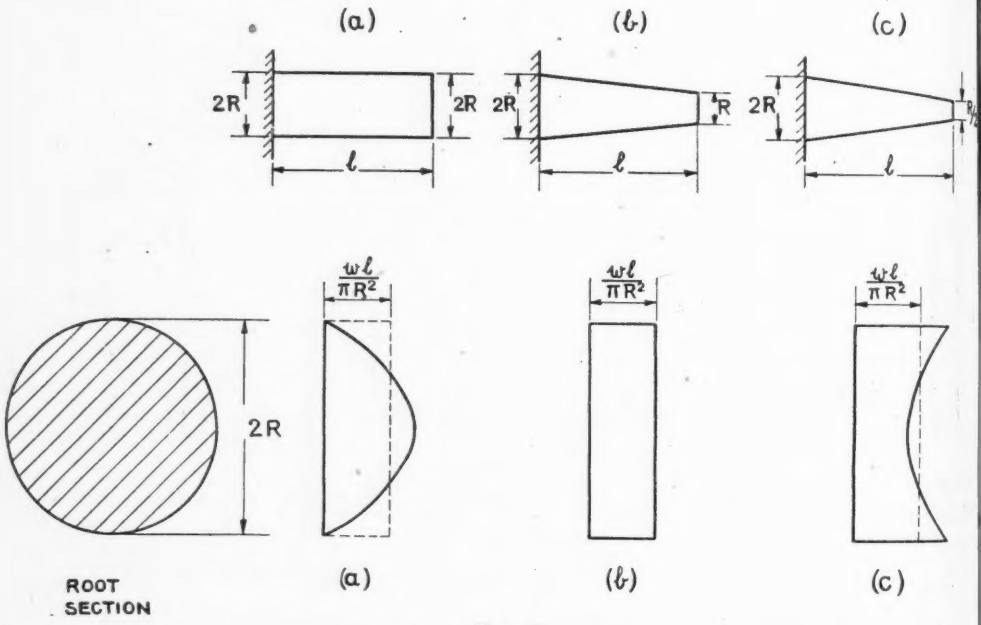


Fig. 6.

Shear stress distributions for conical section beams at the root sections.

Using these formulæ the curves of Fig. 5 have been plotted to show the shear stress distribution for each of the three taper ratios. It will be seen that the taper only slowly changes the distribution, the web of the tapered box section beam still carrying most of the shear.

§5. Conical Beam.

Considerable care is required in calculating the shear stress in a conical beam by the application of the equilibrium equation (1). A direct formula can only be

obtained if the cone angle α is small, when the shear stress, as shown in the Appendix, is given by

$$q=\left(\frac{dM}{dx}\right)\left(\frac{8}{6\pi R^4}\right)(r^2-y_1^2)+M\tan\alpha\left(\frac{8}{6\pi r^4}\right)\left\{\frac{(4y_1^2-r^2)}{r}\right\} \quad . \quad (7)$$

where r is the radius of the beam section considered.

By applying (7) to the three uniformly loaded conical beams of Fig. 6, which have the same root section but in their length l taper from a radius R to R, $\frac{1}{2}R$ and $\frac{1}{4}R$ respectively, the shear stress curves of Fig. 6 have been obtained for the root section, for which equation (7) reduces to

$$q=wl\left(\frac{8}{6\pi R^4}\right)\left\{(R^2-y_1^2)+\left[\frac{(n-1)}{2n}\right](4y_1^2-R^2)\right\} \quad . \quad . \quad (8)$$

where the overall diameter at the root is n times that at the tip. It will be noticed that the shear stress distribution varies with taper very much as in the case of the rectangular beam.

§6. Practical Conclusions.

The following working conclusions are drawn from the foregoing analysis and examples:—

(1) The engineer's method can be readily extended to allow for taper in practical forms of beam.

(2) This extension of the engineer's method gives results in agreement with those of classical theory for certain rectangular and I-section beams.

(3) The effect of taper is practically restricted to the introduction of a change of shear stress distribution proportional to the bending moment at the section concerned.

(4) Negative taper—*i.e.*, taper involving decreasing depth in a region of increasing bending moment—is the most likely to give rise to high shear stresses in practice.

(5) Slight positive taper tends to produce uniformity in the distribution of shear stress in a beam.

It is hoped that the curves of Figs. 3, 5 and 6 will provide an indication of when the investigation of the effects of taper on shear stress should receive special attention in practice.

Appendix

Worked Example.

§1. The example of this appendix relates to a beam of conical form as shown in Fig. 7. No numerical work is given but the shear stress for any point in the beam section is worked out in general terms.

§2. Considering the "slice" indicated in Fig. 7, we have

Width of section at height y, x from the apex $o=2\,((r^2-y^2)^{\frac{1}{2}}$,

Width of section at height y, $(x+dx)$ from $o=2\,(R^2-y^2)^{\frac{1}{2}}$.

For the equilibrium of the block B,

$$q\,\{(R^2-y_1^2)^{\frac{1}{2}}+(r^2-y_1^2)^{\frac{1}{2}}\}\,dx=\int_{y_1}^{R}2\,(R^2-y_1^2)^{\frac{1}{2}}\,dy\,\frac{M+dM}{(\pi/4)\,R^4}y-\int_{y_1}^{r}2\,(r^2-y^2)^{\frac{1}{2}}\,dy\,\frac{M}{(\pi/4)\,r^4}y$$

$$=\left\{\frac{8\,(M+dM)}{3\pi R^4}\right\}(R^2-y_1^2)^{3/2}-\left(\frac{8M}{3\pi r^4}\right)(r^2-y_1^2)^{3/2}=\left(\frac{8dM}{3\pi R^4}\right)(R^2-y_1^2)^{3/2}$$

$$+\frac{8M}{3\pi R^4 r^4}\{r^4\,(R^2-y_1^2)^{3/2}-R^4\,(r^2-y_1^2)^{3/2}\} \quad . \quad . \quad . \quad (9)$$

where M is the bending moment at the section distant x from o.

Now, provided α is small,

$$R = r + dx \tan \alpha,$$
$$R^2 = r^2 + 2rdx \tan \alpha,$$
$$R^4 = r^4 + 4r^3 dx \tan \alpha.$$

Hence, by substituting for R and dividing through by $(r^2 - y_1^2)\,dx$,

$$q\left\{2 + \frac{rdx \tan \alpha}{r^2 - y_1^2}\right\} = \frac{dM}{dx}\,\frac{8}{3\pi r^4}\,\frac{(r^2 - y_1^2)}{(1 + 4dx \tan \alpha / r)}\left\{1 + \frac{3rdx \tan \alpha}{r^2 - y_1^2}\right\}$$
$$+ \frac{M}{dx}\,\frac{8}{3\pi r^4}\,\frac{(r^2 - y_1^2)}{(1 + 4dx \tan \alpha / r)}\left\{1 + \frac{3rdx \tan \alpha}{r^2 - y_1^2} - 1 - \frac{4dx \tan \alpha}{r}\right\}$$

$$\therefore q = \frac{r^2 - y_1^2}{2(r^2 - y_1^2) + rdx \tan \alpha}\left\{\frac{dM}{dx}\,\frac{8}{3\pi r^4}\,\frac{r(r^2 - y_1^2)}{(r + rdx \tan \alpha)}\,\frac{r^2 - y_1^2 + 3rdx \tan \alpha}{(r^2 - y_1^2)}\right.$$
$$\left. + M \tan \alpha\,\frac{8}{3\pi r^4}\,\frac{(4y_1^2 - r^2)}{(r + 4dx \tan \alpha)}\right\}$$

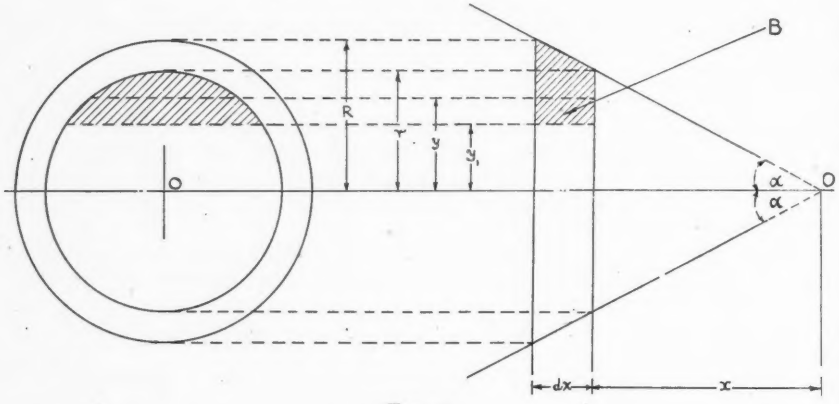


FIG. 7.

Conical beam example.

In the limit, when $dx \to 0$,

$$q = \left(\frac{dM}{dx}\right)\left(\frac{8}{6\pi r^4}\right)(r^2 - y_1^2) + M \tan \alpha \left(\frac{8}{6\pi r^4}\right)\frac{(4y_1^2 - r^2)}{r} \quad . \quad . \quad (10)$$

from which the shear stress q may be calculated from the local shear force dM/dx and bending moment M.

LIST OF REFERENCES.

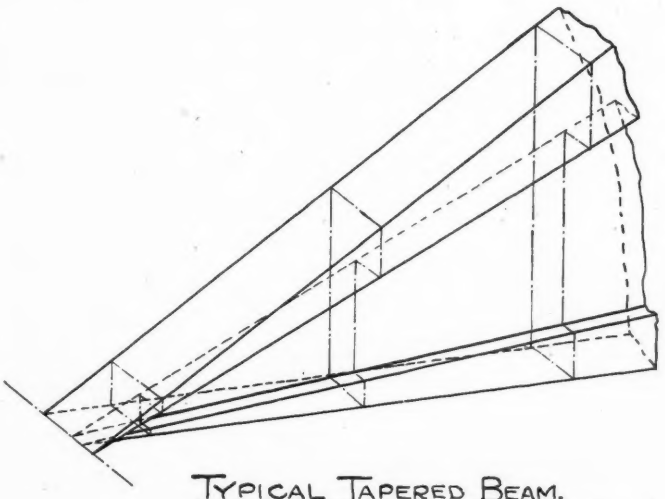
1. A. Morley. Strength of materials. (Longmans 1932.)
2. E. G. Coker and L. N. G. Filon. Treatise on photo-elasticity. (Cambridge University Press 1931.)

TAPERED BEAMS.

By E. H. Atkin, A.F.R.Ae.S.

In a previous paper (*Aircraft Engineering*, November and December, 1938) the writer gave an account of proposed methods of calculating the stresses in tapered girders of a special type. The beam was assumed to have a web of constant thickness which could be dealt with as a two-dimensional stress system, while the booms were assumed to be concentrated lines of material having no flexural rigidity.

It was this latter assumption which appeared to represent one of the main limitations of the theory, so an attempt has been made to remove this limitation; a solution has been found, for the case of tapering booms at any rate, which puts the theory at least on the same level of rigour as the well known theory for parallel beams.



TYPICAL TAPERED BEAM. FIG. 1.

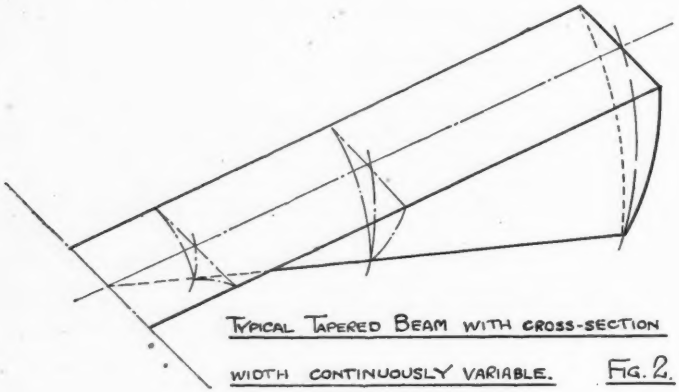
The basic formulæ applicable to two-dimensional stress systems will be found in the writer's previous paper, while the complete theory may be found in various standard works.

The present analysis applies to any arrangement of wedges each of constant thickness continuously attached to each other to form a tapering beam. Such a beam is shown in Fig. 1. It follows that in the limit the analysis can be extended to a girder of continuously variable cross-section, imagined to be made up of an indefinitely large number of elementary edges.

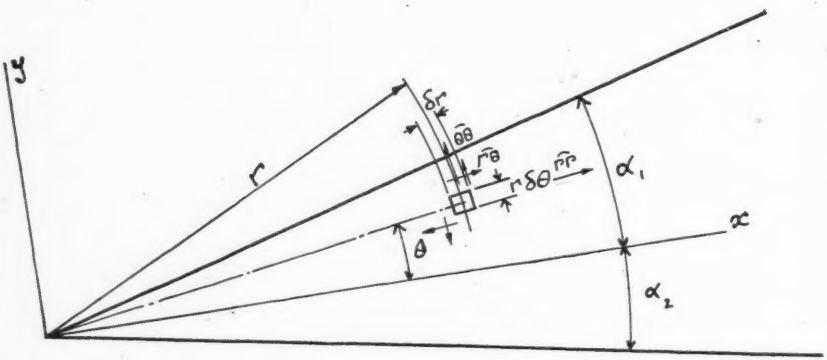
The main condition will then be that for each value of θ (see Figs. 1 and 3) the thickness must be constant for all values of r. Such a girder is shown in Fig. 2.

This paper will be mainly confined to a tapered girder made up of a *finite* number of wedges.

The case of the beam composed of a finite number of wedges and subjected to an apex moment will be considered first.



TYPICAL TAPERED BEAM WITH CROSS-SECTION WIDTH CONTINUOUSLY VARIABLE. FIG. 2.



SYSTEM OF COORDINATES.

FIG. 3.

NOTATION.

$\widehat{rr}$ = radial direction stress.

$\widehat{\theta\theta}$ = circumferential direct stress.

$\widehat{r\theta}$ = shear stress.

X = stress function defining the stresses by the following relations:—

$$\widehat{rr} = \left(\frac{1}{r}\right)\left(\frac{dX}{dr}\right) + \left(\frac{1}{r^2}\right)\left(\frac{d^2X}{d\theta^2}\right)$$

$$\widehat{\theta\theta} = \frac{d^2X}{dr^2}$$

$$\widehat{r\theta} = -(d/dr)\{(1/r)(dX/d\theta)\}$$

U_r = radial displacement.

U_θ = circumferential displacement.

The A_r, B_r and C_r are constants to be determined for each of the n wedges forming the beam.

t_1, $t_2 - t_r - t_{n-1}$, t_n are the thicknesses of the wedges making up the beam.

σ = Poisson's ratio.

M = apex moment.

X and Y = components of apex shear.

It will be found that the following stress function is adequate to deal with the rth wedge of the beam if the entire beam is subjected to a pure apex moment.

$$X_r = A_r \cos 2\theta + B_r \sin 2\theta + C_r\theta \quad . \quad . \quad . \quad . \quad (1)$$

The reader may refer to Fig. 3 for the system of co-ordinates.

On the interface between contiguous wedges there are certain conditions which must be satisfied for all values of r, viz. :—

(a) Equality of resultant shear traction between the rth and $r+1$th wedges of briefly

$$t_r(\widehat{r\theta}_r)_{\theta=r+1} = t_{r+1}(\widehat{r\theta}_{r+1})_{\theta=r+1} \quad . \quad . \quad . \quad . \quad (2)$$

(b) Equality of radial strains and displacements. As $\theta\theta$ is zero, this is equivalent to equality of radial stresses, or

$$(\widehat{rr}_r)_{\theta=r+1} = (\widehat{rr}_{r+1})_{\theta=r+1} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(c) Equality of circumferential displacements or

$$(U_{\theta,\,r})_{\theta=r+1} = (U_{\theta,\,r+1})_{\theta=r+1} \quad . \quad . \quad . \quad . \quad (4)$$

(d) In addition on the extreme radial bounding faces

$$\left.\begin{aligned}(\widehat{r\theta}_1)_{\theta=\alpha_1} &= 0\\ \text{and } (\widehat{r\theta}_n)_{\theta=\alpha_{n+1}} &= 0\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Thus for a beam made up of n wedges of different thicknesses, there are $3n$ disposable constants in the n stress functions, while there are $3(n-1)$ conditions at the interfaces and two further extreme boundary conditions giving in all $3n-1$ conditions in the form of linear homogeneous equations.

(e) The last condition which completes the solution is the equation of the moment of the resultant shears on any circumferential section to the applied moment, or

$$M = r^2 \sum_1^n t_r \int_{\theta r}^{\theta r+1} \widehat{r\theta}\, d\theta \quad . \quad . \quad . \quad . \quad (6)$$

Using the known formulæ, conditions (a), (b), (c), (d) and (e) expressed in detail are as follows :—

From (a)

$$\begin{aligned}&t_r(-2A_r \sin 2\theta_{r+1} + 2B_r \cos 2\theta_{r+1} + C_r)\\ &\quad - t_{r+1}(-2A_{r+1} \sin 2\theta_{r+1} + 2B_{r+1} \cos 2\theta_{r+1} + C_{r+1}) = 0 \quad . \quad . \quad (7)\end{aligned}$$

for all values of r from 1 to $n-1$.

From (b)

$$A_r \cos 2\theta_{r+1} + B_r \sin 2\theta_{r+1} = A_{r+1} \cos 2\theta_{r+1} + B_{r+1} \sin 2\theta_{r+1} \quad . \quad (8)$$

for all values of r from 1 to $n-1$.

From (c)

$$\begin{aligned}&-(-2A_r \sin 2\theta_{r+1} + 2B_r \cos 2\theta_{r+1} + C_r) + \{1/(1+\sigma)\}(-4A_r \sin 2\theta_{r+1} + 4B_r \cos 2\theta_{r+1})\\ &\quad = -(-2A_{r+1} \sin 2\theta_{r+1} + 2B_{r+1} \cos 2\theta_{r+1} + C_{r+1})\\ &\quad + \{1/(1+\sigma)\}(-4A_{r+1} \sin 2\theta_{r+1} + 4B_{r+1} \cos 2\theta_{r+1}) \quad . \quad . \quad (9)\end{aligned}$$

for all values of r from 1 to $n-1$.

And from (d)

$$-2A_1 \sin 2\theta_1 + 2B_1 \cos 2\theta_1 + C_1 = 0 \quad . \quad . \quad . \quad . \quad (10)$$

$$-2A_n \sin 2\theta_{n+1} + 2B_n \cos 2\theta_{n+1} + C_n = 0 \quad . \quad . \quad . \quad (11)$$

Condition (*e*) expressed in detail leads to the following relationship:—

$$M=\sum_{1}^{n} t_r\left[A_r(\cos 2\theta_{r+1}-\cos 2\theta_r)+B_r(\sin 2\theta_{r+1}+\sin 2\theta_r)+C_r(\theta_{r+1}-\theta_r)\right] \quad (12)$$

Besides these conditions there are two others which, from the nature of the case, are satisfied identically.

These conditions are implied in the statement that the apex load must be zero, because any two orthogonal components of the apex load must vanish separately for all values of r.

That this is so is evident from the consideration that we have a completely consistent solution in which there are no body forces and external forces other than the actions at the apex and, therefore, that the resultant actions must be independent of r. This can only be so if there is a moment only at the apex; if there were a resultant force at a cross-section it would vary inversely as r, which is impossible.

In the case of the simpler sections made up of not more than, say, three rectangles, it is not worth while attempting to extract a set of general expressions for the solution.

It is much more direct to substitute immediately in the unmodified equations given above.

There is, however, one step towards an explicit solution which is useful; take equations (8) and (9) pair by pair. The results for each pair are as follows:—

$$\left.\begin{aligned} A_r+\tfrac{1}{2}\{(1+\sigma)/(1-\sigma)\}C_r\sin 2\theta_{r+1}&=A_{r+1}+\tfrac{1}{2}\{(1+\sigma)/(1-\sigma)\}C_{r+1}\sin 2\theta_{r+1}\\ -B_r+\tfrac{1}{2}\{(1+\sigma)/(1-\sigma)\}C_r\cos 2\theta_{r+1}&=-B_{r+1}+\tfrac{1}{2}\{(1+\sigma)/(1-\sigma)\}C_{r+1}\cos 2\theta_{r+1}\end{aligned}\right\} \quad (13)$$

Writing the complete sequence of equations in full and adding or subtracting as necessary it is seen that

$$\left.\begin{aligned} B_{r+1}&=B_1+\tfrac{1}{2}\{(1+\sigma)/(1-\sigma)\}\sum_{1}^{r}(C_{s+1}-C_s)\cos 2\theta_{s+1}\\ A_{r+1}&=A_1-\tfrac{1}{2}\{(1+\sigma)/(1-\sigma)\}\sum_{1}^{r}(C_{s+1}-C_s)(-1)^{s+1}\sin 2\theta_{s+1}\end{aligned}\right\} \quad (14)$$

for all appropriate values of S.

Relations (14) together with (10) and (11) enable complete substitutions to be made in (7) and (12) for the C_r coefficients.

In the case of sections symmetrical about a "neutral axis," the number of unknowns is, of course, immediately halved, and the function for the rectangular element symmetrical about the "neutral axis" becomes an even function.

NUMERICAL EXAMPLE.

Take the section of Fig. 4.

The stress functions may be taken as:—

$$\left.\begin{aligned} X_1&=A_1\cos 2\theta+B_1\sin 2\theta+C_1\theta\\ X_2&=A_2\cos 2\theta+B_2\sin 2\theta+C_2\theta\end{aligned}\right\} \quad . \quad . \quad . \quad (15)$$

In this example it is assumed that θ for each wedge has its zero on the lower bounding line of the corresponding wedge.

The various conditions give rise to the following equations:—

Conditions (*a*) and (*d*)

$$\left.\begin{aligned} -2A_1\sin 2\theta_1+2B_1\cos 2\theta_1+C_1&=0\\ t_1(2B_1+C_1)&=t_2(-2A_2\sin 2\theta_2+2B_2\cos 2\theta_2+C_2)\\ 2B_2+C_2&=0\end{aligned}\right\} \quad (16)$$

Conditions (*b*) and (*c*)

$$\left.\begin{aligned} A_1&=A_2\cos 2\theta_2+B_2\sin 2\theta_2\\ -(2B_1+C_1)+\{4B_1/(1+\sigma)\}&=-(-2A_2\sin 2\theta_2+2B_2\cos 2\theta_2+C_2)\\ &\quad+\{4/(1+\sigma)\}(-A_2\sin 2\theta_2+4B_2\cos 2\theta_2)\end{aligned}\right\}$$

Substituting the values given in Fig. 4 in these relations we have,

$$-0.069799\,A_1+1.998782\,B_1+C_1=0$$

$$10\,B_1+5\,C_1=-0.1854102\,A_2+0.5706339\,B_2+0.3\,C_2$$
$$2B_2+C_2=0$$
$$A_1=0.9510565\,A_2+0.309017\,B_2$$
$$1.2\,B_1-C_1=-0.3708204\,A_2+1.1412678\,B_2-C_2$$

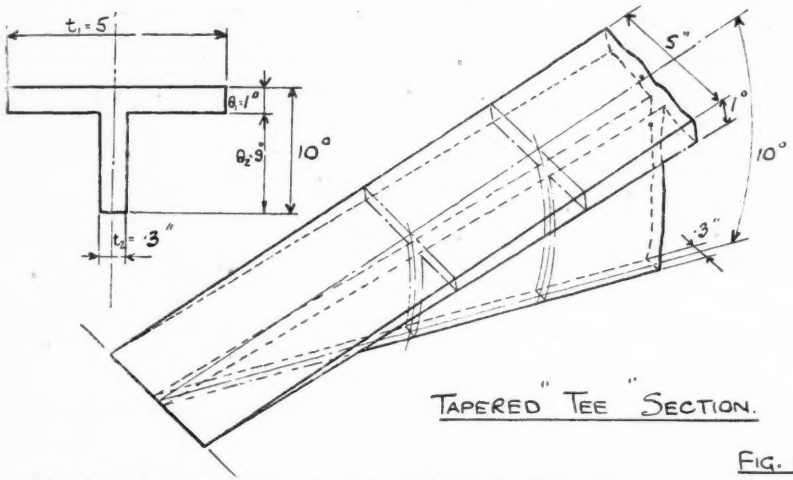
Solving
$$\left.\begin{array}{l} A_1=-0.212259\,A_2 \\ B_2=-3.76457\,A_2 \\ B_1=-0.1274\,A_2 \\ C_2=\ \ 7.52914\,A_2 \\ C_1=\ \ 0.239827\,A_2 \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The final condition (e) is

$$M=t_1\left[A_1(\cos 2\theta_1-1)+B_1\sin 2\theta_1+C_1\theta_1\right]$$
$$+t_2\left[A_2(\cos 2\theta_2-1)+B_2\sin 2\theta_2+C_2\theta_2\right]$$

Which on substitution gives:—

$$A_2=-\frac{M}{0.029586} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$



TAPERED "TEE" SECTION.

FIG. 4.

This finally on substitution in (17) produces the following:—

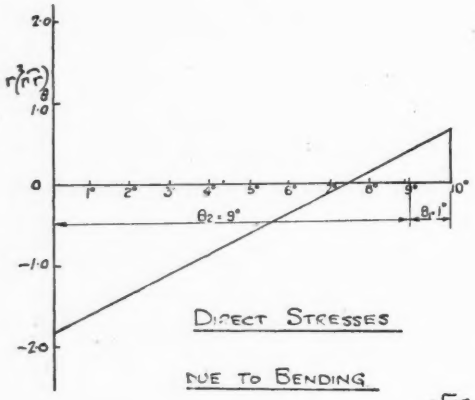
$$\left.\begin{array}{l} A_1=\ \ 7.1743\,M \\ A_2=-\ 33.800\,M \\ B_1=\ \ 4.3061\,M \\ B_2=\ 127.277\,M \\ C_1=-\ \ 8.10613\,M \\ C_2=-254.484\,M \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

All the formulæ can now be stated with numerical coefficients. It will be sufficient to consider the stress distributions. The expressions for these are now:—

$$\left.\begin{array}{l} (\widehat{rr})_{\theta_1}=\dfrac{M}{r^2}(14.3486\cos 2\theta_1+8.6122\sin 2\theta_1) \\ (\widehat{rr})_{\theta_2}=\dfrac{M}{r^2}(-67.6000\cos 2\theta_2+254.554\sin 2\theta_2) \\ (\widehat{r\theta})_{\theta_1}=\dfrac{M}{r^2}(-14.3486\sin 2\theta_1+8.6122\cos 2\theta_1-8.10613) \\ (\widehat{r\theta})_{\theta_2}=\dfrac{M}{r^2}(67.6\sin 2\theta_2+254.554\cos 2\theta_2-254.484) \\ \qquad \text{in which } 0<\theta_1<1^\circ \text{ and } 0<\theta_2<9^\circ \end{array}\right\} \quad . \quad (20)$$

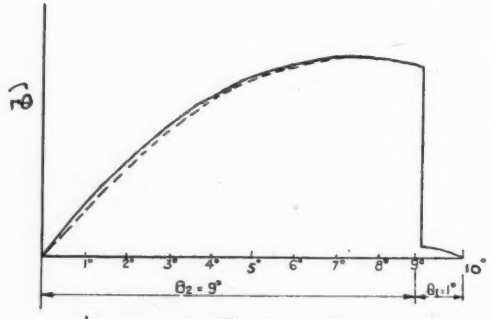
The plotted values of these stresses are given in Figs. 5 and 6 and show that while the direct stress distribution is similar to that given by the "engineers" theory of bending for a parallel beam, there is an induced shear stress system of the same order as the direct stress system which has no analogy in the usual theory for the bending of parallel beams. This point was brought out in my previous paper.

The above analysis can be easily converted into a form suitable for dealing with sections in which the thickness varies continuously in any given circumferential section.



DIRECT STRESSES DUE TO BENDING

FIG. 5.



INDUCED SHEAR STRESSES DUE TO BENDING. FIG. 6.

Under these conditions the constants A_r, B_r and C_r become functions of θ defined by the following simultaneous differential equations:—

$$\{ 4/(1+\sigma) \} (1/\sin 2\theta) (dA/d\theta) = -\{ 4/(1+\sigma) \} (1/\cos 2\theta) (dB/d\theta)$$
$$= -\{ 2/(1+\sigma) \} (dC/d\theta) = (1/t) (dt/d\theta) (-2A \sin 2\theta + 2B \cos 2\theta + C)$$

where t is the thickness for a given θ.

In general this set of equations is extremely intractible; analytical solutions can be obtained for small angles in the case of certain simple sections, but in general a numerical step-by-step method appears to be the only way if an accurate solution is required.

Usually, however, the following artifice will be sufficiently accurate.

It is evident that the induced shear stress distribution must, when added together round a circumferential section, be equal and opposite to the integrated shear component of the radial stresses.

Further, providing the angle of taper is small, the shape of the shear stress distribution will be very closely the same shape as that corresponding to an applied shear on a parallel beam according to the ordinary engineers theory.

The procedure is therefore as follows:—

(*a*) Take a typical cross-section and determine the "constants" in the usual way.

(*b*) Determine the radial stress system by the engineers theory of bending.

(*c*) Integrate the shear components of the radial system as given by (*b*). This gives the internal shear.

(*d*) Calculate the ordinary shear—distribution curve for the section and adjust the scale to give the internal shear (*c*). The solution is then complete. The approximate curve for the example taken has been added in Fig. 6, and shows how closely the two methods agree. (Approximate curve shown dotted.)

APEX LOAD.

Passing on now to the case of a load at the apex, it will be found that the following series of stress functions will satisfy the various conditions:—

$$X_r = r\theta\,(A \sin\theta + B_r \cos\theta) \quad . \quad . \quad . \quad . \quad (21)$$

These represent purely radial direct stress systems.

On the interface between contiguous wedges there are, as in the case of the moment at the apex, certain conditions to satisfy.

These are:—

(*a*) Equality of radial stresses and strains, or briefly, between the rth and $r+1$th wedges.

$$(\widehat{rr}_r)_{\theta=\theta_{r+1}} = (\widehat{rr}_{r+1})_{\theta=\theta_{r+1}} \quad . \quad . \quad . \quad . \quad (22)$$

(*b*) Equality of circumferential displacements, or

$$(U_{\theta,\,r})_{\theta=\theta_{r+1}} = (U_{\theta,\,r+1})_{\theta=\theta_{r+1}} \quad . \quad . \quad . \quad . \quad (23)$$

For a beam made up of n wedges there are $2(n-1)$ such relations (*a*) and (*b*) each relation (*b*) in equivalent to two conditions, and in addition there are the two resultant load conditions for the two components of the apex load.

Hence there are just sufficient relations to determine the $2n$ arbitrary constants A_r, B_r.

The resultant loan conditions are as follows:—

$$\left.\begin{aligned} X &= r\sum_1^n t_r \int_{\theta_r}^{\theta_{r+1}} \widehat{rr}_r \sin\theta\, d\theta \\ Y &= r\sum_1^n t_r \int_{\theta_r}^{\theta_{r+1}} \widehat{rr}_r \cos\theta\, d\theta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (24)$$

By the aid of the known expressions for the stresses in terms of X_r the relation (*a*) expressed in detail is as follows:—

$$A_r \cos\theta_{r+1} - B_r \sin\theta_{r+1} = A_{r+1}\cos\theta_{r+1} - B_{r+1}\sin\theta_{r+1} \quad . \quad . \quad (25)$$

The expression in detail of relation (*b*) calls for the application of the two functions X_r and ϕ_r.

The stress function X_r has already been stated and it may be shown that

$$\begin{aligned}\phi_r = {} & 2A_r\{(\sin\theta)(\log r/r) - (\theta\cos\theta)/r\} \\ & + 2B_r\{(\cos\theta)(\log r/r) + (\theta\sin\theta)/r\} + C_r r \quad . \quad (26)\end{aligned}$$

Whence

$$2\mu U_{\theta,r} = -(1/r)(\partial X/\partial\theta)\,r + \{1/(1+\sigma)\}\,r^2(\partial\phi_r/\partial r) \quad . \quad . \quad (27)$$

$$= -[A_r(\sin\theta + \theta\cos\theta) + B_r(\cos\theta - \theta\sin\theta)]$$
$$+ \{1/(1+\sigma)\}\{2A_r[\sin\theta(1-\log r) + \theta\cos\theta]$$
$$+ 2B_r[\cos\theta(1-\log r) - \theta\sin\theta]$$
$$+ \{(1-\sigma)/(1+\sigma)\}\,C_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

(where C_r represents a rigid body displacement normal to the boundary radius).

And condition (b) becomes

$$-[A_r(\sin\theta_{r+1} + \theta_{r+1}\cos\theta_{r+1}) + B_r(\cos\theta_{r+1} - \theta_{r+1}\sin\theta_{r+1})]$$
$$+ \{1/(1+\sigma)\}\{2A_r[\sin\theta_{r+1}(1-\log r) + \theta_{r+1}\cos\theta_{r+1}]$$
$$+ 2B_r[\cos\theta_{r+1}(1-\log r) - \theta_{r+1}\sin\theta_{r+1}]\} + \{(1-\sigma)/(1+\sigma)\}\,Cr$$
$$= -[A_{r+1}(\sin\theta_{r+1} + \theta_{r+1}\cos\theta_{r+1}) + B_{r+1}(\cos\theta_{r+1} - \theta_{r+1}\sin\theta_{r+1})]$$
$$+ \{1/(1+\sigma)\}\{2A_{r+1}[\sin\theta_{r+1}(1-\log r) + \theta_{r+1}\cos\theta_{r+1}]$$
$$+ 2B_{r+1}[\cos\theta_{r+1}(1-\log r) - \theta_{r+1}\sin\theta_{r+1}]\} + \{(1-\sigma)/(1+\sigma)\}\,C_{r+1} \quad (29)$$

for each dividing radius which when divided into its two parts, by equating the parts independent of r, and coefficients of $\log r$, give the following equations on putting $\theta = \theta_{r+1}$.

$$\left.\begin{aligned} &A_r(\sin\theta_{r+1} + \theta_{r+1}\cos\theta_{r+1}) + B_r(\cos\theta_{r+1} - \theta_{r+1}\sin\theta_{r+1}) + C_r \\ &= A_{r+1}(\sin\theta_{r+1} + \theta_{r+1}\cos\theta_{r+1}) + B_{r+1}(\cos\theta_{r+1} - \theta_{r+1}\sin\theta_{r+1}) + C_{r+1} \\ &A_r\sin\theta_{r+1} + B_r\cos\theta_{r+1} = A_{r+1}\sin\theta_{r+1} + B_{r+1}\cos\theta_{r+1} \end{aligned}\right\} . \quad (30)$$

Solving equations (25) and (30) and substituting it is found that

$$A_r = A_{r+1},\ B_r = B_{r+1},\ \text{and}\ C_r = C_{r+1}.$$

So that

$$\left.\begin{aligned} A_1 = A_2 \ldots A_r \ldots A_{n-1} &= A_n \ldots = A \\ B_1 = B_2 \ldots B_r \ldots B_{n-1} &= B_n \ldots = B \\ C_1 = C_2 \ldots C_r \ldots C_{n-1} &= C_n \end{aligned}\right\} \quad . \quad . \quad . \quad (31)$$

This result checks up with what would otherwise be inferred from the solution for a simple rectangular tapered beam, *i.e.*, that one stress function is applicable to the complete composite cross-section.

The C_r can, therefore, be ignored and the only conditions are (24) relating A and B to the apex load components X and Y.

These equations fully expressed are now:—

$$X = 2\sum_1^n t_r \int_{\theta r}^{\theta r+1} (A\cos\theta - B\sin\theta)\sin\theta\, d\theta$$

$$Y = 2\sum_1^n t_r \int_{\theta r}^{\theta r+1} (A\cos\theta - B\sin\theta)\cos\theta\, d\theta$$

$$\text{or } X = \sum_1^n t_r \{A(\tfrac{1}{2}\sin 2\theta_{r+1} - \tfrac{1}{2}\sin 2\theta_r) + A(\theta_{r+1} - \theta_r)$$
$$+ B(\tfrac{1}{2}\cos 2\theta_{r+1} - \tfrac{1}{2}\cos 2\theta_r\} \quad . \quad . \quad . \quad . \quad (32)$$

$$Y = \sum_1^n t_r \{-A(\tfrac{1}{2}\cos 2\theta_{r+1} - \tfrac{1}{2}\cos 2\theta_r) - B(\theta_{r+1} - \theta_r)$$
$$+ B(\tfrac{1}{2}\sin 2\theta_{r+1} - \tfrac{1}{2}\sin 2\theta_r\} \quad . \quad . \quad . \quad . \quad (33)$$

These equations (32) and (33) represent the complete solution. An example may now be considered.

EXAMPLE.

In order to allow of comparison the same section as was taken in Part I will be considered.

On substitution in equations (32) and (33) we obtain :—

$$\left.\begin{aligned} X &= 0.258245\,A - 0.035751\,B \\ Y &= 0.035751\,A - 0.005480\,B \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (34)$$

So that the results may be compared with the " engineers theory " of bending, the components X and Y will be chosen to give a resultant normal to the neutral plane as usually defined.

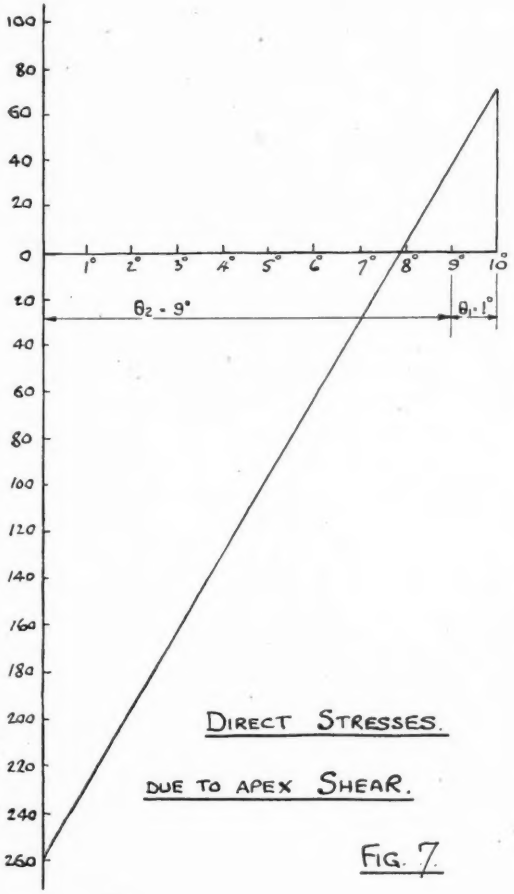


FIG. 7.

If the resultant is R then under these conditions

$$X = -0.1345627\,R$$
$$Y = 0.9909051\,R$$

Solving for A and B

$$A = -\ 261.1\,R$$
$$B = -1885\,R$$

and therefore

$$\widehat{rr} = -(2R/r)\,(261.1\cos\theta - 1885\sin\theta) \quad . \quad . \quad . \quad (35)$$

The plotted result is given in Fig. 7.

From this diagram it is seen that for small angles of taper, the result is the same as obtained by the ordinary theory. As soon as the taper becomes appreciable, however, it is evident that the stress distribution will no longer be linear.

There is no difficulty in extending equations (32) and (33) so that they apply to a cross-section of continuously varying thickness.

This is done by looking upon the t_r in equations (32) and (33) as a function of θ.

On doing this the modified equations are seen to be:—

$$X = 2\int_0^\alpha t\,(A\cos\theta - B\sin\theta)\cos\theta d\theta \quad . \quad . \quad . \quad (32a)$$

$$Y = 2\int_0^\alpha t\,(A\cos\theta - B\sin\theta)\sin\theta d\theta \quad . \quad . \quad . \quad (32b)$$

It is only necessary to carry these integrations when the angle of taper is appreciable.

GENERAL.

It is hoped that the preceding work will help towards a clearer insight into the mechanism of the stresses in tapered beams.

The analysis given is at least as rigorous as the engineer's theory for parallel beams, the wrong application of which to tapered beams can now be seen to lead to entirely erroneous results.

ELEMENTARY MATHEMATICS.

H. Levy, M.A., D.Sc., F.R.S.E. T. Nelson and Sons, Ltd. 1942. 5/- net.

This is a further volume in Nelson's Aero Science Manuals of which three have been published and reviewed in the Journal. These manuals are all under the general editorship of Professor Levy and an excellent job of work he has made of them. If the remaining books in this series are anything up to the standard of those already published students of the future can congratulate themselves that much of the horror of mathematics, in particular, will have gone. Education, in this country, has for years been a favourite object of attack and very rightly so. The reviewer, for example, was taught arithmetic from a certain standard text book and twenty-five years afterwards he found his son was being taught from the same book, and was meeting the same difficulties and facing the same misunderstandings. There was the same attitude of mind of the teacher. The boy who couldn't understand arithmetic from this text book was obviously backward. In the reviewer's case he ultimately took an honours degree in mathematics, and all his life has hated arithmetic and still is unable to add and multiply with the certainty his answers will be correct.

Professor Levy in just over 200 pages teaches the elements of Arithmetic, Algebra and Trigonometry, makes us familiar with the use of graphs and gives us a glimpse on the last page, of the meaning of integral calculus. His statements are clear and concise and he leads us up the mathematical garden in such an interesting way that we have no difficulty in realising that $p+s+d$ may equally stand for pigs plus sheep plus donkeys or for pansies plus syringas plus delphiniums. So algebraic symbols appear side by side with arithmetical ones, with the astonishing result that when we came to the chapter entitled "First Steps in Algebra," we do not worry at all to find the expansion of $(x+y)^n$ given and understanding it, and above all not being puzzled or frightened by it. That kind of thing is an accomplishment of which the significance is not likely to be realised by teachers brought up in the old school. The whole method of educational approach has been so badly in need of revision for the last two generations that one welcomes the wedges which Professor Levy and his follow authors in this series are driving into the structure. It is to be hoped they will split that structure wide open so that a fresh method of teaching will be evolved which will interest, instruct and amuse the student, instead of only instructing. And how the younger generation always hate the bare meatless bones of instruction. Professor Levy is to be heartily congratulated and one looks forward to his Advanced Mathematics in the same series.

WINGS OVER OLYMPUS.

By T. H. Wisdom. Allen and Unwin, Ltd. 1942. 9/-.

This is a book of war-flying with a difference—the difference being that the author knows how to write. We had already discovered this, with a feeling of surprised relief, before reading on the cover page that he was a professional journalist. It is a story of the war in the Middle East, though not, as the title would imply, exclusively of the all-too-brief Greek campaign. "We saw the silvery light of the night sky at Heliopolis, and the brazen sunrise over Mersa Matruh; we passed the halcyon days at Menidi, and welcomed the spring at Paramythia, the 'valley of fairy tales'; we sweated together in the desolation of Shaiba, and roughed it in battle-scarred, fly-blown El Adem; we took off from

earthquake-shattered Larissa, and we camped in the olive groves of Maleme." This sentence from the prologue is a very good summary of the book, and it is borne out by a profusion of beautiful, sunlit photographs, taken in the air and on the ground, which no one who loves the Mediterranean can see without a feeling of nostalgia. There is plenty of action, as well as an appreciation of beauty—tales of the author's friends who fought, and only too often died, in the air above the age-old battlefields of Greece, and there is one fine story that symbolises the spirit of that indomitable country. " An evzone, a member of the proud Highland regiment of Greece, had been ordered to haul down the blue and white flag of Greece from the Acropolis, and hoist in its place the hated black and red swastika rag of modern Germany. He had refused. His Commanding Officer, the tears streaming down his face, ordered him to do it. The evzone, in the khaki kilt and tunic that had replaced the quilted white kilt and blue and gold tunic, the fine ceremonial uniform of the King's Guard, pulled down Greece's flag for the last time. He wrapped himself in it and dived to his death on the rocks below the Acropolis."

FUNDAMENTALS OF VIBRATION STUDY.

By R. G. Manley. Chapman and Hall. 1942. 13s. 6d. net.

In aviation the study of vibration problems has rapidly become of greater importance than in most other branches of engineering. This study has been forced on aircraft engineers by the number of operational troubles in civil aircraft due to vibration, and in military aircraft even more so with increasing speeds and increasing engine powers.

The modern aeroplane is, unfortunately, not wholly in the hands of one man or a few men in its design. Instruments, armaments and accessories have to be fitted in to a design according to operational requirements, with the result that the machine as it finally flies may reveal unexpected vibration troubles which could have been foreseen if the control had been in the same hands throughout. The vibration engineer will inevitably have to form one of the team of designers and be given the problem as a whole finally instead of in bits and pieces which is what so often happens now.

The published literature on vibration has tended to make many people think it is a job for the tame mathematician, and such a book as the one under review has been badly needed. It is a book founded on the subject matter of a course of lectures delivered by the author to members of the Vibration Department of the de Havilland Company.

The first two chapters deal with systems having one degree of freedom, in undamped and damped motion. Chapter III is concerned with undamped motion with two degrees of freedom and Chapter IV with more than two degrees of freedom. In this chapter the author very rightly draws attention to the unfortunate use of the terms mechanical impedance and dynamic modulus for the ratio torque/displacement. He, himself, uses the term dynamic stiffness, which is analogous to static stiffness. It is hoped this term will be adopted as it has a clear physical meaning and the use of dynamic stiffnesses provides the easiest way of calculating the resonant frequencies of systems with large damping forces. The experience of the author himself has evidently led to the change in nomenclature as in November, 1941, an article in the Journal of the Society by him was published with the title, " Mechanical Impedance of Damped Vibrating Systems," in which he acknowledged the term had been used by his company since 1939.

Chapter V covers the vibration of heavy shafts and beams and the last chapter the important Fourier Analysis method. Three appendices on Pure and Applied Mathematics and Frequency equations and numerical solutions form a valuable addition to Mr. Manley's book.

This is a most useful introduction to Vibration problems. It is indeed more than an introduction and it is hoped that it will lead the engineer who has hitherto avoided the subject, or turned it on to the " tame mathematician," to take it up

and avoid in the early stages of design many vibration problems which have, in the past, led to the belief that a particular aircraft has proved " difficult " on trials. The early consideration of such problems, contemporaneously with the general design of the aircraft, will lead to a considerable saving of time and hair, and Mr. Manley's book opens the road for saving both.

JANE'S ALL THE WORLD'S AIRCRAFT, 1941.

Edited by Leonard Bridgman. Sampson, Low, London. 1942. Three Guineas net.

It takes something more than a world war to stop the publication of Jane. Its editors may come and go, censors may wrack their wills on it and paper controllers may hesitate about but not refuse the paper required for a publication which has become more necessary than ever. It is remarkable, indeed, how fully informed the editor is of world progress in aviation, and it is comforting to note that over 70 pages are devoted to Civil Aviation.

As usual, " All the World's Aircraft " is divided into four main sections, Service and Civil Aviation and Aeroplanes and Aero Engines. An introduction has been written by Sir Charles Bruce-Gardner, Chairman of the Society of British Aircraft Constructors. Sir Charles, in his review of the years 1940-41, points out how a short-sighted air policy was only overcome by a long-sighted industry which by bitter struggle kept going despite the meagre orders of the Government. There was, for example, he notes, the manager who took orders for corinthian bagatelle boards to employ men in his experimental section. That it should have risen to one of the most powerful industries in this country in such a short space of time is entirely due to the foresight of a few men, not the least of whom are those who steered the industry through those times when, if not exactly dying, it was certainly on a bed of sickness. Sir Charles is clearly an advocate for the big bomber policy. He pays a well-deserved tribute to American co-operation, technically as well as from a production point of view.

In the Historical (Service Aviation) Section a useful innovation is the printing of national markings, a very necessary piece of information at the present time.

There is an excellent summary of the air war in 1941 over Great Britain, the Atlantic, Germany, the Middle East and elsewhere, which may put into better perspective for many people what overwhelming demands must be made upon the aircraft industry and how efficiently they are facing up to these demands.

The Historical (Civil Aviation) Section reveals an astonishing volume of civil aviation activities despite the world-wide character of the war. It is comforting reading, for example, to know that new routes are continually being opened up, and one reads that the Isle of Man Services, Ltd., maintains a regular three daily service between Liverpool and the Isle of Man and for the twelve months September, 1940-August, 1941, carried over 12,000 passengers, one and a quarter of a million pounds of mail and over 30,000 pounds of freight, while there are a number of flourishing services to Ireland. Canada, Australia and India are keeping the civil aviation flag flying in the British Empire, but it is when we turn to the activities of the United States that it is realised that here civil aviation is more than holding its own. During the first six months of 1941 over a million and a half paying passengers were carried by American lines, an increase of 29 per cent. in passenger revenue miles over the corresponding period of 1940.

In the Aeroplane Section a welcome innovation is the presentation of outline general drawings of most of the important aircraft. This section is excellently done and is a survey of the military air effort of the Allies and the Axis which is surprisingly up to date.

The Aero Engine Section reveals little that has not been known of development save the significant fact that horse-powers are rising. The development of the high powered engine in every country is one, the importance of which cannot be over-exaggerated.

BLOCKADE BY AIR.

By J. M. Spaight. Geoffrey Bles. 1942. 10/6.

Fifteen years ago, when amiable idealists at Geneva were discussing how best to reprove any nation that might be so ungentlemanly as to threaten war, Mr. Spaight was already calling attention to our national danger. In a world hypnotised by the League of Nations idea, he dared to say straight out: " A predatory nation, a Napoleon, could brush the whole flimsy fabric of pains and penalties aside. . . . Something has happened in our generation to destroy what hope there ever was that a sanction-wielding world league could prevent aggression and ensure peace. That ' something ' is the coming of air power." (" Pseudo-Security," 1928.)

Events have justified Mr. Spaight, and it is to his credit that he refrains from saying so in this new book, which is written in the fine spirit of a fighting Irishman. The first part deals with what has actually been done by our " Air Corsairs "—as the author picturesquely terms the Coastal Command aircraft who are attacking enemy merchant shipping; the second outlines a policy for future development.

Some interesting facts are given in the preliminary retrospect. Few people can have realised that nearly a hundred British ships were sunk by air attack during the Spanish Civil War. It is satisfactory to know that, in the second six months of 1941 the actual tonnage of enemy shipping sunk by our naval and air forces was greater than that of our shipping sunk by the Axis, although our enemies had far fewer ships afloat. Total sinkings of British, Allied and neutral shipping from 1939 to the end of 1941 are estimated as about 8,600,000 tons; of enemy ships, between five and six million tons.

Mr. Spaight ends with a plea for a " scorched sea policy," to bring Europe to its knees by starvation. " We must go all out for the scorching of the narrow seas. We must establish a veritable reign of terror in them. That statement will shock some people, but this is totalitarian war and the old kid-glove methods must go." " Blockade, bombing, buccaneering . . . will have made a vital contribution to the overthrow of the gangster powers."

A book to be read by all—to be marked, learned and inwardly digested by those who have the control of Britain's naval and aerial policy.

INTERNATIONAL AIR TRANSPORT AND NATIONAL POLICY.

Oliver J. Lissitzyn. Foreword by Edward B. Warner. Council on Foreign Relations, New York. 1942. $5.00.

There are two great aviation dramas. In the theatre of war we are watching, on a world-wide stage, the most thrilling drama yet produced. In the theatre of peace, in the drama of civil aviation, it is the interval. We look back on Act I, with its touches of romance, its spots of sordidness, its commercial rivalries, its suppressed flashes of vision and its exposure of British parsimony. We wonder—is this a tragedy with Act II continuing the grim story till the time comes for another sinister interval and an Act III of final collapse? Or is this a two-act morality with the first Act showing the perils of improvising and the second the rewards of planning? We hope the latter.

It must be confessed that Act II is still in the draft stage. The authors are many, their scenarios various. Soon they must pool their ideas and produce a plan for the renaissance of civil aviation. Some of them seem to want to write the wrong Act II; others see civil aviation, not as a mass of competing nations and companies, striving in a political war for air supremacy, but as the working out of a coherent world-wide plan.

To all new-world planners, new dealers and Utopians there is one bit of elementary advice to be given—study history. Occasionally history will indicate how history should be made. More often, it will indicate how it should not, which is almost as useful. Mr. Lissitzyn's book is a valuable contribution to

civil aviation history and should be studied by all those concerned with the future of civil aviation. It is not a story book, to be read at a sitting, but a massive compilation of facts, statistics, and events. It is in fact a competent review of Act I, and seems to accept as natural and probably enduring the competitive, commercial, political and national atmosphere of the past. Mr. Lissitzyn concludes (page 421) that " Internationalisation of air transport is not likely to come without internationalisation of military and political power." This is probably correct, but we shall never get internationalisation of anything—and we infer he, like most of us, believes internationalisation to be " a good thing "—unless we start to internationalise something. And in civil aviation, so completely will the old order have disappeared by the time the war ends, we have a particularly fine chance of starting again on the right lines. To your reviewer the chief lesson from the book, which adequately exposes the inefficient rivalries and cloaked motives of the pre-war order, is that the nations of the world should, in the general clean-up after this war, co-ordinate the world's air routes as a first step towards changing civil aviation from an instrument of national policy to a service to humanity.

In recent years the U.S.A. has produced a number of historians of noteworthy calibre. They have a genius for painstaking research and their work is notable for the orderly marshalling of numerous facts. Mr. Lissitzyn joins this good company. The sheer volume of his data has been implied above. Their arrangement is good. He writes from an American point of view which, as he is an American writing the third study in American Foreign Relations for the American Council on Foreign Relations, is what we expect him to do. He looks at his subject, International Air Transport and National Policy, from all angles. He explains why States must be interested in air transport, the economic factors in State aid, the nature of Governmental control, and how Governments have promoted (positively and negatively) air transport. He discusses monopolies and competition, the diplomacy of air transport and the freedom of the air. If one can offer a criticism it is that the large canvas has been filled in with so much detail that the basic structure of the picture has become a little obscured. To the eye straining for a sight of the future, two chapters are of especial interest—Chapter VI, a summary on national interest and national policy, and Chapter XVI, on the freedom of the air. In the earlier chapter the author sees recent developments in transportation and communication tending to produce political entities of greater and greater size, the smaller states being absorbed in the larger, so that the world of to-morrow may be a world of a few highly centralised great powers. He sees the continued decline of national boundaries as barriers to intercourse and correspondingly an increase in international strains. Following this line of speculation he visualises the few surviving great empires warring or preparing to war for power, and as a result the military aspect of aviation emphasised. The next step in this pessimistic vision is of a world order established through war or through conference, but a world order more likely than not of a singularly unpleasant kind. The impression one is left with is of another exposition of the doctrine of survival of the fittest and the author suggests as a reason for striving for superior means of transportation, that it may lead a state to the winning instead of the losing side. Mr. Lissitzyn does not say this *will* be the course of events, but he thinks it might. It may be unjust to suggest that he thinks it more likely than not. We should be grateful to him in any case for painting an unpleasant picture and giving us thereby further stimulus to produce a better.

In the latter chapter referred to above, the author leads up to the famous French proposal to the League of Nations in 1932, for internationalisation by the creation of " International Air Transport Unions," one for each of the great continental areas of the world. The Americans and the British were, with the Germans and Italians, in opposition to this. It is time that at any rate the

Americans and British begin to think about it again. International civil aviation cannot, as Mr. Lissitzyn suggests, co-exist with contention between nations. But are we not now learning our lesson? Can we not, when the present great drama, or ghastly farce, or what you will, is played out, plan for national cultures and international economics and communications? If we cannot, then not only civil aviation, but civilisation, so much wrapped up with it and nearly spelt the same, is doomed.